Solving Problems with a Small K_{eq}

* Some equilibrium expressions may require you to use the quadratic equation to solve

Using Quadratic Equation

* Reminder

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

Example

- If initial concentration of N_2O_4 is 0.50 M, what are the equilibrium concentrations if Keq is 0.0059?
  - $N_2O_{4(g)} \rightleftharpoons 2\ NO_{2(g)}$

Solution

* Fill out an ICE table. Write a chemical equation and add know values.

	$N_2O_{4(g)}$ $\rightleftharpoons$	$2\ NO_{2(g)}$
Initial	0.50	0
Change	-x	+2x
Equilibrium	0.50 - x	+2x

Solution

* Now use the K_{eq} expression to solve

$$K_{eq} = \frac{[HI]^2}{[H_2][I_2]}$$

$$0.0059 = \frac{(2x)^2}{(0.5 - x)}$$

Since the equation doesn't involve a perfect square you must use the quadratic formula

Solution

* Rearrange into the quadratic equation

$0.0059\ (0.5 - x) = (2x)^2$
$0.00295 - 0.0059x = 4x^2$
$-4x^2 - 0.0059x + 0.00295 = 0$
$-1(4x^2 + 0.0059x - 0.00295) = 0$

This mean a = 4, b= 0.0058, c = -0.00295

Solution

* Now substitute into the quadratic equation:

$$x = \frac{-0.0058 \pm \sqrt{[(0.0058)^2 - 4(4)(-0.00295)]}}{2(4)}$$

x = 0.026 OR x = - 0.0029

Since x can't be negative, x = 0.026

Solution

* Fill out an ICE table. Write a chemical equation and add know values.

$$N_2O_{4(g)} \rightleftharpoons 2\ NO_{2(g)}$$

Initial	0.50	0
Change	-x	+2x
Equilibrium	0.50 - 0.026	+2 (0.026)

Solution

* Fill out an ICE table. Write a chemical equation and add know values.

$$N_2O_{4(g)} \rightleftharpoons 2\ NO_{2(g)}$$

Initial	0.50	0
Change	-x	+2x
Equilibrium	0.474	0.052

Approximating Small Quantities

* When dealing with small concentration changes, you can sometimes assume that initial and final concentrations are the same.

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Example:
[]i = 0.065
Change is [] = -0.000032

[]eq= 0.065 - 0.000032 = 0.0064968

Approximating Small Quantities

* "hundred rule" is an assumption made to simplify problems
* If the ratio of [initial concentration of reactant]/K > 100, than x is very small compared to initial concentration and may be discarded from some calculations.

Example

* In a reaction k is 4.2×10^{-8} in the equilibrium constant. A chemist puts 0.085 mol of nitrogen and 0.038 mol into a 1.5 L container. Determine the equilibrium concentration of NO.

* $N_{2(g)} + O_{2(g)} \rightleftharpoons 2NO_{(g)}$

Solution

* First calculate the concentration of each product and reactant:

$$[N_3] = \frac{n}{V} = \frac{0.085\ mol}{1.5L} = 0.057\ mol/L$$

$$[O_2] = \frac{n}{V} = \frac{0.038\ mol}{1.5L} = 0.025\ mol/L$$

Solution

* Set up an ICE table and record information

$$N_{2(g)} + O_{2(g)} \rightleftharpoons 2NO_{(g)}$$

	$N_{2(g)}$	$O_{2(g)}$	$2NO_{(g)}$
Initial	0.057	0.025	0
Change	-x	-x	+2x
Equilibrium	0.057-x	0.025 - x	2x

Solution

* Check to see if “hundred rule applies”
* []i/K >100
* $0.57/(4.2 \times 10^{-8})$ = >100

This means you can assume the concentrations will remain approximately the same

Solution

* Now substitute and solve

$$K_{eq} = \frac{[NO]^2}{[N_2][O_2]}$$

$$4.2 \times 10\text{-}8 = \frac{(2x)^2}{(0.057)(0.025)}$$

$$4x^2 = 5.985 \times 10^{-11}$$

$$x = 3.9 \times 10^{-6}$$

Solution

* Set up an ICE table and record information

$N_{2(g)} + O_{2(g)} \rightleftharpoons 2NO_{(g)}$

	$N_{2(g)}$	$O_{2(g)}$	$2NO_{(g)}$
Initial	0.057	0.025	0
Change	-x	-x	+2x
Equilibrium	0.057-x	0.025 - x	$2(3.9 \times 10^{-6})$

Solution

* Set up an ICE table and record information

$$N_{2(g)} + O_{2(g)} \rightleftharpoons 2NO_{(g)}$$

	$N_{2(g)}$	$O_{2(g)}$	$2NO_{(g)}$
Initial	0.057	0.025	0
Change	-x	-x	+2x
Equilibrium	0.057-x	0.025 - x	7.8×10^{-6}

Homework

* p 457 #71, 72