# Math and Chemistry

## Significant Digits

All measurements involve uncertainty. One source of this uncertainty is the measuring device itself. Another source is your ability to perceive and interpret a reading. In fact, you cannot measure anything with complete certainty. The last (farthest right) digit in any measurement is always an estimate.

The digits that you record when you measure something are called *significant digits*. Significant digits include the digits that you are certain about, *and* a final, uncertain digit that you estimate. Follow the rules below to identify the number of significant digits in a measurement.

### Rules for Determining Significant Digits

**Rule 1** All non-zero numbers are significant.
- 7.886 has four significant digits.
- 19.4 has three significant digits.
- 527.266 992 has nine significant digits.

**Rule 2** All zeros that are located between two non-zero numbers are significant.
- 408 has three significant digits.
- 25 074 has five significant digits.

**Rule 3** Zeros that are located to the left of a measurement are *not* significant.
- 0.0907 has three significant digits: the 9, the third 0 to the right, and the 7.

**Rule 4** Zeros that are located to the right of a measurement may or may not be significant.
- 22 700 may have three significant digits, if the measurement is approximate.
- 22 700 may have five significant digits, if the measurement is taken carefully.

When you take measurements and use them to calculate other quantities, you must be careful to keep track of which digits in your calculations and results are significant. Why? Your results should not imply more certainty than your measured quantities justify. This is especially important when you use a calculator. Calculators usually report results with far more digits than your data warrant. Always remember that calculators do not make decisions about certainty. You do. Follow the rules given below to report significant digits in a calculated answer.

### Rules for Reporting Significant Digits in Calculations

**Rule 1 Multiplying and Dividing**
The value with the fewest number of significant digits, going into a calculation, determines the number of significant digits that you should report in your answer.

**Rule 2 Adding and Subtracting**
The value with the fewest number of decimal places, going into a calculation, determines the number of decimal places that you should report in your answer.

**Rule 3 Rounding**
To get the appropriate number of significant digits (rule 1) or decimal places (rule 2), you may need to round your answer.
- If your answer ends in a number that is greater than 5, increase the preceding digit by 1. For example, 2.346 can be rounded to 2.35.
- If your answer ends with a number that is less than 5, leave the preceding number unchanged. For example, 5.73 can be rounded to 5.7.
- If your answer ends with 5, increase the preceding number by 1 if it is odd. Leave the preceding number unchanged if it is even. For example, 18.35 can be rounded to 18.4, but 18.25 is rounded to 18.2.

**Sample Problem**

**Using Significant Digits**

**Problem**

Suppose that you measure the masses of four objects as 12.5 g, 145.67 g, 79.0 g, and 38.438 g. What is the total mass?

**What Is Required?**

You need to calculate the total mass of the objects.

**What Is Given?**

You know the mass of each object.

**Plan Your Strategy**
- Add the masses together, aligning them at the decimal point.
- Underline the estimated (farthest right) digit in each value. This is a technique you can use to help you keep track of the number of estimated digits in your final answer.
- In the question, two values have the fewest decimal places: 12.5 and 79.0. You need to round your answer so that it has only one decimal place.

**Act on Your Strategy**

$$\begin{array}{r} 12.\underline{5} \\ 145.6\underline{7} \\ 79.\underline{0} \\ +\ 38.43\underline{8} \\ \hline 275.\underline{608} \end{array}$$

Total mass = 275.608 g

Therefore, the total mass of the objects is 275.6 g.

**Check Your Solution**

- Your answer is in grams. This is a unit of mass.
- Your answer has one decimal place. This is the same as the values in the question with the fewest decimal places.

Add/subtract/divide FIRST, round second

**Practice Problems**

**Significant Digits**

1. Express each answer using the correct number of significant digits.
   a) 55.671 g + 45.78 g = 101.4 g
   b) 1.9 mm + 0.62 mm = 2.5 mm
   c) 87.9478 L − 86.25 L 1.698 L
   d) 0.350 mL + 1.70 mL + 1.019 mL = 3.13 mL
   e) 5.841 cm × 6.03 cm = 35.2 cm
   f) $\frac{17.51\ g}{2.2\ cm^3}$ = 8.0 g/cm³

## Scientific Notation

One mole of water, $H_2O$, contains 602 214 199 000 000 000 000 000 molecules. Each molecule has a mass of 0.000 000 000 000 000 000 000 029 9 g. As you can see, it would be very awkward to calculate the mass of one mole of water using these values. To simplify large numbers when reporting them and doing calculations, you can use scientific notation.

**Step 1** Move the decimal point so that only one non-zero digit is in front of the decimal point. (Note that this number is now between 1.0 and 9.99999999.) Count the number of places that the decimal point moves to the left or to the right.

**Step 2** Multiply the value by a power of 10. Use the number of places that the decimal point moved as the exponent for the power of 10. If the decimal point moved to the right, exponent is negative. If the decimal point moved to the left, the exponent is positive.

6.02 000 000 000 000 000 000 000.

23 21 18 15 12 9 6 3

$6.02 \times 10^{23}$

**Figure D.1** The decimal point moves to the left.

0.000 000 000 000 000 000 000 02.9 9 g

3 6 9 12 15 18 21 23

$2.99 \times 10^{-23}$

**Figure D.2** The decimal point moves to the right.

Figure D.3 shows you how to calculate the mass of one mole of water using a scientific calculator. When you enter an exponent on a scientific calculator, you do not have to enter (×10).

| Keystrokes | Display |
|---|---|
| 6 . 0 2 EXP 2 3 | 6.02 23 |
| × 2 . 9 9 EXP 2 3 ± | 2.99 −23 |
| = | 17.998 |

Round to three significant digits and express in scientific notation: $1.80 \times 10^1$ g/mol

**Figure D.3** On some scientific calculators, the EXP key is labelled EE. Key in negative exponents by entering the exponent, then striking the ± key.

### Rules for Scientific Notation

**Rule 1** To multiply two numbers in scientific notation, add the exponents.

$(7.32 \times 10^{-3}) \times (8.91 \times 10^{-2})$
$= (7.32 \times 8.91) \times 10^{(-3 + -2)}$
$= 65.2212 \times 10^{-5}$
$\rightarrow 6.52 \times 10^{-4}$

**Rule 2** To divide two numbers in scientific notation, subtract the exponents.

$(1.842 \times 10^6\ g) \div (1.0787 \times 10^2\ g/mol)$
$= (1.842 \div 1.0787) \times 10^{(6-2)}$
$= 1.707611 \times 10^4\ g$
$\rightarrow 1.708 \times 10^4\ g$

**Rule 3** To add or subtract numbers in scientific notation, first convert the numbers so they have the same exponent. Each number should have the same exponent as the number with the greatest power of 10. Once the numbers are all expressed to the same power of 10, the power of 10 is neither added nor subtracted in the calculation.

$(3.42 \times 10^6\ cm) + (8.53 \times 10^3\ cm)$
$= (3.42 \times 10^6\ cm) + (0.00853 \times 10^6\ cm)$
$= 3.42853 \times 10^6\ cm$
$\rightarrow 3.43 \times 10^6\ cm$

$(9.93 \times 10^1\ L) - (7.86 \times 10^{-1}\ L)$
$= (9.93 \times 10^1\ L) - (0.0786 \times 10^1\ L)$
$= 9.8514 \times 10^1\ L$
$\rightarrow 9.85 \times 10^1\ L$

Practice problems are given on the following page.

**Practice Problems**

### Scientific Notation

1. Convert each value into correct scientific notation.
   - **(a)** 0.000 934 $9.34 \times 10^{-4}$
   - **(b)** 7 983 000 000 $7.983 \times 10^{9}$
   - **(c)** 0.000 000 000 820 57 $8.2057 \times 10^{-10}$
   - **(d)** $496 \times 10^6$ $4.96 \times 10^{8}$
   - **(e)** $0.000\ 06 \times 10^1$ $6.0 \times 10^{4}$
   - **(f)** $309\ 72 \times 10^{-8}$ $3.0972 \times 10^{-4}$
2. Add, subtract, multiply, or divide. Round off your answer, and express it in scientific notation to the correct number of significant digits.
   - **(a)** $(3.21 \times 10^{-3}) + (9.2 \times 10^2)$ $9.2 \times 10^{2}$
   - **(b)** $(8.1 \times 10^3) + (9.21 \times 10^2)$ $9.0 \times 10^{3}$
   - **(c)** $(1.010\ 1 \times 10^1) - (4.823 \times 10^{-2})$ $1.005 \times 10^{1}$
   - **(d)** $(1.209 \times 10^6) \times (8.4 \times 10^7)$ $1.0 \times 10^{14}$
   - **(e)** $(4.89 \times 10^{-4}) \div (3.20 \times 10^{-2})$ $1.53 \times 10^{-2}$

## Logarithms

Logarithms are a convenient method for communicating large and small numbers. The *logarithm*, or "log," of a number is the value of the exponent that 10 would have to be raised to, in order to equal this number. Every positive number has a logarithm. Numbers that are greater than 1 have a positive logarithm. Numbers that are between 0 and 1 have a negative logarithm. Table D1 gives some examples of the logarithm values of numbers.

**Table D.1** Some Numbers and Their Logarithms

| Number | Scientific notation | As a power of 10 | Logarithm |
|---|---|---|---|
| 1 000 000 | $1 \times 10^6$ | $10^6$ | 6 |
| 7 895 900 | $7.8590 \times 10^5$ | $10^{5.8954}$ | 5.8954 |
| 1 | $1 \times 10^0$ | $10^0$ | 0 |
| 0.000 001 | $1 \times 10^{-6}$ | $10^{-6}$ | −6 |
| 0.004 276 | $4.276 \times 10^{-3}$ | $10^{-2.3690}$ | −2.3690 |

Logarithms are especially useful for expressing values that span a range of powers of 10. The Richter scale for earthquakes, the decibel scale for sound, and the pH scale for acids and bases all use logarithmic scales.

### Logarithms and pH

The pH of an acid solution is defined as $-\log[H_3O^+]$. (The square brackets mean "concentration.") For example, suppose that the hydronium ion concentration in a solution is 0.0001 mol/L ($10^{-4}$ mol/L). The pH is $-\log(0.0001)$. To calculate this, enter 0.0001 into your calculator. Then press the [LOG] key. Press the [±] key. The answer in the display is 4. Therefore, the pH of the solution is 4.

There are logarithms for all numbers, not just whole multiples of 10. What is the pH of a solution if $[H_3O^+] = 0.004\ 76$ mol/L? Enter 0.00476. Press the [LOG] key and then the [±] key. The answer is 2.322. This result has three significant digits—the same number of significant digits as the concentration.

**CONCEPT CHECK**

For logarithmic values, only the digits to the right of the decimal point count as significant digits. The digit to the left of the decimal point fixes the location of the decimal point of the original value.

What if you want to find $[H_3O^+]$ from the pH? You would need to find $10^{-pH}$. For example, what is $[H_3O^+]$ if the pH is 5.78? Enter 5.78, and press the [±] key. Then use the [$10^x$] function. The answer is $10^{-5.78}$. Therefore, $[H_3O^+]$ is $1.7 \times 10^{-6}$ mol/L.

Remember that the pH scale is a negative log scale. Thus, a decrease in pH from pH 7 to pH 4 is an increase of $10^3$, or 1000, in the acidity of a solution. An increase from pH 3 to pH 6 is a decrease of $10^3$, or 1000, in acidity.

**Practice Problems**

### Logarithms

1. Calculate the logarithm of each number. Note the trend in your answers.
   - **(a)** 1 **(b)** 5 **(c)** 10 **(d)** 50 **(e)** 100 **(f)** 500 **(g)** 50 000 **(h)** 100 000
2. Calculate the antilogarithm of each number.
   - **(a)** 0 **(b)** 1 **(c)** −1 **(d)** 2 **(e)** −2 **(f)** 3 **(g)** −3
3. **(a)** How are your answers for question 2, parts (b) and (c), related?
   - **(b)** How are your answers for question 2, parts (d) and (e), related?
   - **(c)** How are your answers for question 2, parts (f) and (g), related?
   - **(d)** Calculate the antilogarithm of 3.5.
   - **(e)** Calculate the antilogarithm of −3.5.
   - **(f)** Take the reciprocal of your answer for part (d).
   - **(g)** How are your answers for parts (e) and (f) related?
4. **(a)** Calculate log 76 and log 55.
   - **(b)** Add your answers for part (a).
   - **(c)** Find the antilogarithm of your answer for part (b).
   - **(d)** Multiply 76 and 55.
   - **(e)** How are your answers for parts (c) and (d) related?